Midterm Introduction to Logic (AI and MA + Guests)

Wednesday 11 December, 2018, 9:00–11:00 AM

Instructions: Read Carefully

- ☞ Write the course version you are in at the top of the first page (either "AI" or "Math + Guests").
- ☞ Only write your student number at the top of the exam, **not your name**, so that we can grade anonymously. Also put your student number at the top of any additional pages.
- ☞ Put the name of your tutorial group (AI 1, AI 2, ... or MG 1, MG 2, MG 3, MG 4) at the top of the exam and any additional pages.
- ☞ Leave the first ten lines of the first page blank (for the calculation of your grade).
- ☞ Use a blue or black pen (so no pencils, no markers, no red pens).
- ☞ With the regular exercises, you can earn 90 points. By writing your student number and tutorial group on all pages, you earn a first 'free' 10 points. With the bonus exercise, you can earn an additional 10 points. The total exam grade is: *(the number of points you earned with the regular and bonus exercises + the first 'free' 10) divided by 10, with a maximum grade of 10.*

Good Luck!

1: Translation into propositional logic (10 points) Translate the following sentences into *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key — one key for the whole exercise. Represent as much logical structure as possible.

a. Yoshua will understand Gary only if Yoshua read Gary's book or prepared for the debate very well.

b. They will agree to disagree unless neither Gary nor Yoshua is stubborn.

Translation key:

A**:** They will agree to disagree.

G**:** Gary is stubborn.

Y**:** Yoshua is stubborn.

U**:** Yoshua will understand Gary.

B : Yoshua read Gary's book.

P : Yoshua prepared for the debate very well.

Formalizations:

a. $U \to (B \vee P)$

b. $A \vee (\neg G \wedge \neg Y)$; also correct: $A \vee \neg(G \vee Y)$

2: Translation into first-order logic (10 points) Translate the following sentences to *first-order logic*. Do not forget to provide the translation key — one key for the whole exercise. Represent as much logical structure as possible and let the domain of discourse be the set of all people, buildings, locations, and vehicles in Groningen.

a. Dilip prefers the Forum to the Martini Tower, even though he did not vote in favor of the Forum.

b. If the Forum is between the Schuitendiep and the Martini Tower, then Dilip rides to the Forum on his bike precisely if he does not ride to the Martini Tower on his bike.

Translation key:

$\mathsf{Between}(x, y, z)$: x is between y and z.

$\mathsf{Ride}(x, y, z)$: x rides to y on z.

$\mathsf{Prefer}(x, y, z)$: x prefers y to z.

$\mathsf{Voted}(x, y)$: x voted in favor of y.

a: Dilip's bike

b: the Forum

c: the Schuitendiep

d: Dilip

e: the Martini Tower

f: the Forum

Formalizations:

a. $\mathsf{Prefer}(d, f, e) \land \neg\mathsf{Voted}(d, f)$

b. $\mathsf{Between}(f, c, e) \to (\mathsf{Ride}(d, f, a) \leftrightarrow \neg\mathsf{Ride}(d, e, a))$

3: Formal proofs (30 points) Give formal proofs of the following inferences. Do not forget to provide justifications in the correct order. You may only use the Introduction and Elimination rules and the Reiteration rule.

a.
$$\begin{array}{|l} a = b \\ b \neq c \\ \hline a \neq c \end{array}$$

b.
$$\begin{array}{|l} (\neg A \to B) \to \neg A \\ \hline \neg A \end{array}$$

c.
$$\begin{array}{|l} A \lor B \\ C \leftrightarrow B \\ \hline A \lor C \end{array}$$

a.
$$\begin{array}{|ll} 1.\ a = b & \\ 2.\ b \neq c & \\ \hline \quad \begin{array}{|l} 3.\ a = c \\ \hline 4.\ b = c \\ 5.\ \bot \end{array} & \begin{array}{l} \\ = \text{Elim: } 3, 1 \\ \bot \text{ Intro: } 4, 2 \end{array} \\ 6.\ a \neq c & \neg \text{ Intro: } 3\text{–}5 \end{array}$$

b.

1. $(\neg A \to B) \to \neg A$	
2. A	
3. $\neg A$	
4. $\bot$	$\bot$ Intro: 2,3
5. B	$\bot$ Elim: 4
6. $\neg A \to B$	$\to$ Intro: 3–5
7. $\neg A$	$\to$ Elim: 1, 6
8. $\bot$	$\bot$ Intro: 2, 7
9. $\neg A$	$\neg$ Intro: 2–8

c.

1. $A \vee B$	
2. $C \leftrightarrow B$	
3. A	
4. $A \vee C$	$\vee$ Intro: 4
5. B	
6. C	$\leftrightarrow$ Elim: 2, 5
7. $A \vee C$	$\vee$ Intro: 6
8. $A \vee C$	$\vee$ Elim: 1, 3–4, 5–7

4: Truth tables (15 points) Use *truth tables* to answer the next questions. Provide the full truth tables. Order the rows in the truth tables as follows:

P	Q	R	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a = b	b = c	c = a	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a. Is $((P \vee \neg Q) \to (R \leftrightarrow Q)) \to (\neg P \vee (\neg Q \to \neg R))$ a *tautology*?
Do not forget to draw an explicit conclusion from the truth table and to explain your answer.

P	Q	R	$((P$	$\vee$	$\neg Q)$	$\to$	$(R$	$\leftrightarrow$	$Q))$	$\to$	$($	$\neg P$	$\vee$	$(\neg Q$	$\to$	$\neg R))$
T	T	T		T	F	T		T		**T**		F	T	F	T	F
T	T	F		T	F	F		F		**T**		F	T	F	T	T
T	F	T		T	T	F		F		**T**		F	F	T	F	F
T	F	F		T	T	T		T		**T**		F	T	T	T	T
F	T	T		F	F	T		T		**T**		T	T	F	T	F
F	T	F		F	F	T		F		**T**		T	T	F	T	T
F	F	T		T	T	F		F		**T**		T	T	T	F	F
F	F	F		T	T	T		T		**T**		T	T	T	T	T
				(2)	(1)	(3)		(1)		(4)		(1)	(3)	(1)	(2)	(1)

The sentence *is* a tautology, because there are only 'T's under the main connective in column 4.

b. Is the sentence $(\neg(a = b) \land \neg(c = b)) \to \neg(c = a)$ a *logical truth*?
Draw an explicit conclusion, explain your answer and indicate the spurious rows.

$a = b$	$c = b$	$c = a$	(	$\neg$	$(a = b)$	$\land$	$\neg$	$(c = b)$	)	$\to$	$\neg$	$(c = a)$	
T	T	T		F	T	F	F	T		**T**	F	T	
T	T	F		F	T	F	F	T		**T**	T	F	spurious row!
T	F	T		F	T	F	T	F		**T**	F	T	spurious row!
T	F	F		F	T	F	T	F		**T**	T	F	
F	T	T		T	F	F	F	T		**T**	F	T	spurious row!
F	T	F		T	F	F	F	T		**T**	T	F	
F	F	T		T	F	T	T	F		**F**	F	T	
F	F	F		T	F	T	T	F		**T**	T	F	

The sentence *is not* a logical truth because there is an 'F's under the main connective in a non-spurious row, namely in row 7.

5: Normal forms of propositional logic (15 points)

a. Provide a negation normal form (NNF) of this sentence:
$(P \leftrightarrow Q) \to (\neg P \to R)$

$$
\begin{array}{lll}
 & (P \leftrightarrow Q) \to (\neg P \to R) & \\
\Leftrightarrow & \neg(P \leftrightarrow Q) \lor (\neg P \to R) & \text{def } \to \\
\Leftrightarrow & \neg((P \to Q) \land (Q \to P)) \lor (\neg P \to R) & \text{def } \leftrightarrow \\
\Leftrightarrow & (\neg(P \to Q) \lor \neg(Q \to P)) \lor (\neg P \to R) & \text{De Morgan} \\
\Leftrightarrow & (\neg(\neg P \lor Q) \lor \neg(Q \to P)) \lor (\neg P \to R) & \text{def } \to \\
\Leftrightarrow & ((\neg\neg P \land \neg Q) \lor \neg(Q \to P)) \lor (\neg P \to R) & \text{De Morgan} \\
\Leftrightarrow & ((P \land \neg Q) \lor \neg(Q \to P)) \lor (\neg P \to R) & \text{double } \neg \\
\Leftrightarrow & ((P \land \neg Q) \lor \neg(\neg Q \lor P)) \lor (\neg P \to R) & \text{def } \to \\
\Leftrightarrow & ((P \land \neg Q) \lor (\neg\neg Q \land \neg P)) \lor (\neg P \to R) & \text{De Morgan} \\
\Leftrightarrow & ((P \land \neg Q) \lor (Q \land \neg P)) \lor (\neg P \to R) & \text{double } \neg \\
\Leftrightarrow & ((P \land \neg Q) \lor (Q \land \neg P)) \lor (\neg\neg P \lor R) & \text{def } \to \\
\Leftrightarrow & ((P \land \neg Q) \lor (Q \land \neg P)) \lor (P \lor R) & \text{double } \neg
\end{array}
$$

b. Provide a conjunctive normal form (CNF) of this sentence:
$\neg(\neg(\neg(P \to Q) \lor R) \lor P)$

$$
\begin{array}{lll}
 & \neg(\neg(\neg(P \to Q) \lor R) \lor P) & \\
\Leftrightarrow & \neg\neg(\neg(P \to Q) \lor R) \land \neg P & \text{De Morgan} \\
\Leftrightarrow & (\neg(P \to Q) \lor R) \land \neg P & \text{double } \neg \\
\Leftrightarrow & (\neg(\neg P \lor Q) \lor R) \land \neg P & \text{def } \to \\
\Leftrightarrow & ((\neg\neg P \land \neg Q) \lor R) \land \neg P & \text{De Morgan} \\
\Leftrightarrow & ((\neg\neg P \lor R) \land (\neg Q \lor R)) \land \neg P & \text{distributivity} \\
\Leftrightarrow & ((P \lor R) \land (\neg Q \lor R)) \land \neg P & \text{double } \neg
\end{array}
$$

Indicate the intermediate steps. You do *not* have to provide justifications for the steps.

6: Set theory (10 points) Consider the following three sets:

$$A = \{\{\emptyset\}\}, \quad B = \{a, \{\emptyset\}\}, \quad C = \{a\} \quad \text{and} \quad R = \{(a,b),(b,a),(c,c)\}.$$

For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

a. $\emptyset \in A$ No

b. $\emptyset \subseteq A$ Yes

c. $\emptyset \subseteq \{A\}$ Yes

d. $A \cap B = \emptyset$ No

e. $\{A \cap B\} = A$ No

f. $A = B \cup \{\emptyset\}$ No

g. $A = B \setminus C$ Yes

h. R is transitive. No

i. R is reflexive. No

j. R is symmetric. Yes

7: Bonus question (10 points) Give a formal proof of the following inference:

$$\vdash \neg((P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)) \to (\neg P \wedge \neg Q)$$

Do not forget to provide justifications in the correct order. You may only use the Introduction and Elimination rules and the Reiteration rule.

1. $\neg((P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q))$	
2. P	
3. Q	
4. $P \wedge Q$	$\wedge$ Intro: 2,3
5. $(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)$	$\vee$ Intro: 4
6. $\perp$	$\perp$ Intro: 5,1
7. $\neg Q$	$\neg$ Intro: 3-6
8. $P \wedge \neg Q$	$\wedge$ Intro: 2,7
9. $(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)$	$\vee$ Intro 8
10. $\perp$	$\perp$ Intro: 9,1
11. $\neg P$	$\neg$ Intro: 2-10
12. Q	
13. $\neg P \wedge Q$	$\wedge$ Intro: 11,12
14. $(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)$	$\vee$ Intro: 13
15. $\perp$	$\perp$ Intro: 14,1
16. $\neg Q$	$\neg$ Intro: 12-15
17. $\neg P \wedge \neg Q$	$\wedge$ Intro: 11, 16
18. $\neg((P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)) \to (\neg P \wedge \neg Q)$	$\to$ Intro: 1-17